



## ANATOLY KUSRAEV: A SCIENTIST BY BELIEF

S. S. Kutateladze<sup>1</sup>

Accepted: 1 December 2023 / Published online: 30 December 2023  
© The Author(s), under exclusive licence to Springer Nature Switzerland AG 2023



Anatoly Kusraev belongs to the school of Leonid Kantorovich; i.e., Kusraev's research traverses tracts in the general direction of the school. Kantorovich entered science under the influence and guidance of Georgy Fikhtengoltz and Vladimir Smirnov in the first third of the twentieth century. These years brought about the revolutionary changes in mathematics which underwent transformations in the frameworks of the axiomatic method and set-theoretic stance. General topology and functional analysis sprang to life, and all branches of mathematics transformed drastically. Scholars discovered various combinations of mathematical structures and searched for applications to the new problems searched application to the new problems of science and technology.

The supervisor of Kantorovich was Fikhtengoltz, an outstanding professor who rightfully understood the well-known shortcomings of his course in differential and integral calculus which was in fact revealing and influential. It is not accidental that his three-volume textbook was crowned with the appendix “General View of Limit,” whose first sections were devoted to ordered sets.

Synthesizing the ideas of Banach and order, Kantorovich plunged into the development of the theory of semi-ordered vector spaces. He introduced the so-called  $K$ -spaces, i.e., Dedekind complete vector lattices. Kantorovich called these “my spaces” in his private notebooks. The ideas of partial order are related to the problems of economics in which we encounter

---

✉ S. S. Kutateladze  
sskut@math.nsc.ru

<sup>1</sup> Sobolev Institute, Novosibirsk, Russia

multi-dimensional exponents that cannot be compared by using a sole parameter. The relation of order is naturally tied with searching maxima and minima. The simplest here are the problems of linear programming discovered by Kantorovich. Speaking in plain language, the main topic of the Kantorovich school is the theory of linear inequalities. This general theme includes convex and nonsmooth analysis, the theory of Banach lattices, Riesz and Kantorovich spaces, the theory of positive and dominated operators, and many similar branches of mathematics.

Inequality precedes equality, as the mind of a human starts with discerning differences. The next step consists in distinguishing between great and small. That is how order enters mentality. Understanding sameness, an alias of equality, is much harder. Despite this peculiarity, inequality had formally entered mathematics later than equality. “Linear” is older than “convex.” The appearance of the former adjective in the English language is assigned to the 1640s. Although lines are everywhere in Euclid’s *Elements*, the linearity of processes entered mathematics at the Age of Enlightenment with the discovery of the infinitesimal calculus. Differentiation and integration are linear operations, which starts the history of linearity in sciences.

In fact, inequalities were hidden in Euclidean geometry which deals primarily with closed convex figures. However, the intrinsic connections between inequality and convexity became obvious much later. Even the word “convex” belongs to the sixteenth century. *The Oxford English Dictionary* claims that the first appearance of the word occurred in the book of 1571 *A Geometricall Practise Named Pantometria* by Thomas Digges. Stemming from the remote ages, convexity reigns in functional analysis and optimizations. Convexity embraces generation, separation, calculus, and approximation. Generation appears as duality; separation, as optimality; calculus, as representation; and approximation, as stability. Linearity and order lead to ordered vector spaces whose study began in the twentieth century, inaugurating the modern theory of linear inequalities.

As regards convex analysis, Kusraev’s contribution includes the introduction and study of the topological general position of various convex objects, the development of general subdifferentiation and Clarke theory, and the study of some versions of the discrete maximum principle and quasidifferential analysis.

Kusraev has substantially modernized the tool-kit of Kantorovich’s school by using and developing the methods of Boolean valued analysis, i.e., the nonstandard techniques of Boolean valued set-theoretic models. Recall that Eugene Gordon discovered the place of Kantorovich spaces in the modern mathematics. He proved that each universally complete  $K$ -space and only this space represents the field of reals in an appropriate Boolean valued model. Thus,  $K$ -spaces acquired immortality as the new realizations of the real axis, which opens the limitless perspectives of research and demonstrates the indispensability of the concepts of Kantorovich’s school.

Kusraev is an acknowledged leader in the field of Boolean-valued analysis. He owns beautiful results in dominated operator theory, vector integration, the structure of Kaplansky–Hilbert modules,  $C^*$ -modules and operator algebras, spaces with mixed norm, injective Banach lattices, and similar cyclic objects and phenomena that were unknown previously. Kusraev mathematics satisfies the Mac Lane criteria of excellence. Kusraev’s contributions are inevitable, illuminating, deep, relevant, responsive, and timely.

The life of a human is a unique experiment, a sequence of events whose laws of control are hidden. There exist various techniques of recognition, for instance, in cryptology. To split the sequence under study into pieces and compare the latter helps often to grasp what was encrypted. Jubilees are the days of cameral treatment of data and the search of the concealed regularities of the distance traveled.

The human being is egocentric and, hence, egoistic by nature. Birthdays and jubilees are tranquilizers alleviating the horrors and pains of the contradiction between egoism and dependence.

A jubilee is a feast of recognition rather than a rehearsal of a memorial service. Anyone is rarely present at the anniversary of his 100 years. The centennial is the occasion for the new generation to get knowledge of the life lessons of a noble person. Completely different is the jubilee of an alive contemporary. The hero of the day is the only one who knows everything about the hero’s life. The jubilee is his feast, the event when he recognizes how his relatives and friends see him. Kusraev’s and mine worldlines are close about a half-century, and we know and understand much of one another. Pondering in the beginning of January of how to congratulate Kusraev on his 70th birthday, I was notified that Yuri Manin had just passed away. Manin was an outstanding mathematician and thinker of the modern times. Quite a few know that the most important fragment of Kusraev’s creative path is tied up with Manin. It was Manin who acquainted the mathematical community of the Soviet Union with the Cohen’s solution of the first Hilbert problem. Using the recommendation by Kantorovich, Kusraev delivered his first results on Boolean valued analysis at Manin’s seminar. I happened to be present at the first Manin’s talk on the solution of the continuum hypothesis at the Moscow Mathematical Society. I was also

honored to communicate with Manin by letter during the last 15 years. Thinking about Manin and Kusraev, I could not help but understand that both are scientists by belief. It stands to reason to explain this term.

Science is wholesome and indivisible, but it has two hypostases. First of all, science is the system of knowledge and explanation. This is science-1. Also, science is the system of preservation and production of knowledge. This is science-2. Science-1 is an individual, nonclass, nonnational and nonsocial phenomenon. It is irrelevant what persons were Ptolemy, Bacon, Leibniz, Heisenberg, or Petrovsky; what citation indices they had, and how much their salaries or other benefits were. All these circumstances mean nothing to science-1. Science-1 administers truth.

Science-2 is a national and social institution. Science-2 is a servant and dependent of the society. For science-2 it matters who is a student, who is an emiritus professor, or who is a winner of a megagrant. Both sciences coexist in every scientist in this or other country. Science-1 is the same for everyone, whereas science-2 differs from country to country essentially. Science-1 and science-2 should not be confused in neither thin nor thick years.

Science-1 is impossible without a personality, as well as the personality of a scientist is determined by science-1. Science-1 has no room for fanabaria, xenophobia, envy, selfseeking, and rudeness. In the absence of science-1 there are neither problems, nor results, nor school, nor laboratory, and nor institute. Left are just imitations, degrees and ranks, budget grabbing, contests, awards, citation indices, flattery and self-conceit, zeal in appraising cronies, and belly dancing at the government pillars.

Science-2 is depersonalized, which brings nothing but harm to science-1. The present and future giants of science-1 resemble heroes of the past. They are so great that it is impossible to overlook them. They are the first who are attacked by dwarfs, cannibals, and dinosaurs. Sticks are usually put in the wheels of the others, and this is the habit of either those who lack any wheels even in a project or those who feel disgusted when seeing the wheels of others. Human sins enter scientists through science-2.

The imperative of science-1 is the service to truth. David Hilbert formulated this imperative in the *anti-ignorabimus* maxim: “Wir müssen wissen. Wir werden wissen.” The imperative of science-2 is the service to humankind. Aleksandr Alexandrov formulated this imperative as universal humanism. The Russian language distinguishes between the concepts of scientist and science employee. A scientist serves science-1, and a science employee belongs to science-2. The same linguistic difference is reflected in the English terms scholar and scientist.

A scientist by belief is an active thinker who obeys the imperatives of both hypostases of science. It is a school that brings up a scientist by belief. Priority and his place in the hierarchical structure of the scientific community are important but secondary to him. Short-lived is the joy of a scientist by belief when he succeeded in his efforts. He clearly sees the scarcity of his knowledge and the limitations of his tool-kit. Science is tied to failures. A scientist by belief is modest since he understands the miserliness of his own capabilities in comparison with the greatness of the spirit of the human population. Pseudoscience is alien to a scientist by belief. He feels a responsibility not only for his own watch tower but also for the whole realm of science which safeguards a human from the environment and himself.

Kusraev is a scientist by belief who contributes greatly to modern mathematics and substantially enriches the scientific landscape of his native republic, Northern Ossetia-Alania.

Colleagues, friends, students, and coworkers of Anatoly Kusraev wish him, his family, and next of kin long happy years and new achievements in his service to science and people.