GENERALIZED MATHEMATICAL MODEL FOR POPULATION DYNAMICS AND AGE STRUCTURE

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In the domain $\Omega = \{(x,t) : 0 < x < l, 0 < t\}$ consider a mathematical model that describes the dynamics of the population size of a species

$$\partial_{0x}^{\alpha} u(x,t) + \lambda \partial_{0t}^{\beta} u(x,t) + c(x) u(x,t) = f(x,t), \tag{1}$$

with $\alpha = 1$ and $\beta = 1$ it is a generalization for the McKendrick von Foerster equation [1]. Here $u = u(x,t)$ is interpreted as the population density of the age $x \in (0,l)$ at time $t > 0$, $\partial_{0x}^{\alpha} u(x,t)$ — change in the number of individuals of age x at fixed t, $\partial_{0t}^{\beta} u(x,t)$ — change in the number of individuals at different times for a fixed x, $\partial_{0x}^{\alpha}, \partial_{0t}^{\beta}$ is the regularized (Caputo) derivative [2], $0 < \alpha < \beta < 1$, $f(x,t)$, $c(x)$ are the given functions, while the function $c(x)$ is the function of mortality, and $f(x,t)$ describes various demographic processes, $\lambda = \mathrm{const}$ — coefficient change status. For equation (1) the solution of the Cauchy problem was found in [1].

By a *regular solution* of equation (1) in the domain Ω we mean the function $u = u(x,t)$ from the class $u(x,t) \in C(\overline{\Omega})$; $u_x(x,t), \partial_{0x}^{\alpha} u(x,t), \partial_{0t}^{\beta} u(x,t) \in C(\Omega)$ satisfying equation (1) in the domain Ω.

Problem. Find a regular solution $u(x,t)$ to equation (1) in the domain Ω satisfying conditions

$$u(0,t) + \int_0^l M(\xi)\, u(\xi,t)\, d\xi = \varphi(t), \quad 0 < t, \tag{2}$$

$$u(x,0) = \tau(x), \quad 0 \leqslant x \leqslant l. \tag{3}$$

Condition (2) is called the fertility equation [1], where $u(0,t)$ is the is the density of newborn individuals in the population, l is the age limit, $M(\xi)$ is the fertility function, and $\varphi(t)$ is some control function for possible human intervention on population dynamics.

The initial condition (3) is necessary to study the dynamics of the age structure of the population.

In article the existence of a unique regular solution to the problem (1)–(3) is proved.

References

1. *Nakhushev A. M.* Equation of Mathematical Biology.—Moscow: Vysshaja shkola, 1995
2. *Nakhushev A. M.* Fractional Calculus and its Application.—Moscow: Fizmatlit, 2003.—272 p.