A Riesz-Frechet theorem in Riesz spaces

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Abstract

Let F is a Dedekind complete Riesz space with weak order unit and conditional expectation operator T. In addition we assume that T is strictly positive and that F is T-universally complete. We define $E = L^2(T) := \{f \in F \,|\, f^2 \in F\}$, where multiplication as as defined in the f-algebra F_u, the universal completion of F. The T-strong dual of $L^2(T)$ denoted by $\hat{E}$ consists of the maps $\mathfrak{f} : L^2(T) \to R(T) := T(F)$ such that $\mathfrak{f}$ is $R(T)$-homogeneous, order continuous and there exists $k \in R(T)_+$ so that $|\mathfrak{f}(g)| \leq k\|g\|_{T,2}$ for all $g \in L^2(T)$. Here $\|g\|_{T,2} = \sqrt{T(g^2)}$ and the space $\hat{E}$ has $R(T)$ valued norm $\|\mathfrak{f}\| := \inf\{k \in R(T)_+ \,|\, |\mathfrak{f}(g)| \leq k\|g\|_{T,2}$ for all $g \in E\}$. We give a Riesz-Frechet theorem which provides an isometry between E and $\hat{E}$.

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