# ON POLYNOMIAL CONJECTURES OF NILPOTENT LIE GROUPS UNITARY REPRESENTATIONS

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Abstract. Let $G$ be a connected and simply connected nilpotent Lie group of Lie algebra $\mathfrak{g}$, $K$ an analytic subgroup of $G$, $\chi$ a unitary character of $G$ and $\pi$ an irreducible unitary representation of $G$. In this setting, the orbit method allows to identify the unitary dual of $G$ to the space of coadjoint orbits. Using the enveloping algebra of the complexified Lie algebra of $\mathfrak{g}$, we consider two algebras of differential operators $D\pi(G)^K$ and $D_\tau(G/K)$ associated respectively to the restriction $\pi|K$ of $\pi$ to $K$ and to the monomial representation $\tau = \mathrm{Ind}_G^K \chi$. Under the assumption that these representations are of finite multiplicities, the polynomial conjectures stating that $D\pi(G)^K$ and $D_\tau(G/K)$ are $K$-invariant polynomial rings hold. In this lecture, I will overview some history of the conjectures and some restrictive cases. Once restricted to codimension one normal subgroups of $G$, the study of the geometry and the saturation of coadjoint orbits plays a crucial role in the proofs.

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